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Analytical range evaluation for therapeutic protons in stack of materials

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HIGHLIGHTS

- Range straggling, defined as spread in the stopping point of protons, is studied.
- An analytical method for range determination through studying the depth-dose curves is proposed.
- Effect of initial energy distribution on the values of the range is investigated.
- The validity of range-energy approximation for stack of different materials is tested.
- The results are encouraging for accurate dose modeling in tissue inhomogeneities.

ABSTRACT

As one of the most clinically relevant parameters in proton radiotherapy, the range of incident particles can be measured either by counting the number of protons or through depth-dose evaluation in the target. In the latter, the range is defined as the depth in the target at the distal 80% point of the Bragg peak. In this work, a highly accurate analytical model was employed to predict depth-dose distribution, and hence the range, in a desired target. Aiming to study the effect of energy spread on the range, proton beams with initial Gaussian distributions have been considered. For our arbitrary tested energies, the results show that the more the width of energy distribution increases, the more the Bragg peaks shift in depth, by about -0.25% to -25%, compared with those of monoenergetic beams. Furthermore, it was found that for different widths of initial energy spectrum, keeping the mean energy the same, the range remains unchanged. It was also shown that the results corresponding to utilizing analytical range determination for proton beams of different incident energies in stack of materials deviate from those of Monte Carlo simulations by less than 1.7%. The results are encouraging, although accurate modeling of analytical proton dose distribution in the presence of tissue inhomogeneities is still an unsolved problem.

KEYWORDS

Proton radiotherapy
Range
Analytical dosimetry
Bragg curve
Bortfeld model

1 Introduction

The important advantage of the proton radiotherapy compared with conventional neutron and photon therapies is that protons finally stop in matter, and the dose beyond the stopping point is negligible. The depth-dose distribution of such a beam is called the Bragg curve with a sharp peak in depth, known as the Bragg peak. It has a Gaussian-shaped cross section, and highly depends on the proton energy and the target material. The characteristics of the dose distribution is largely determined by the physical properties of the protons, the beam accelerator, and the beam nozzle designs used to control and shape the beam (Lu and Flanz, 2012).

Because of its longitudinally narrow peak, a strictly monoenergetic proton beam is not convenient to be used for treatment of an extended tumor area. An appropriate clinical proton beam requires both spreading to a uniform area in the lateral direction and a uniform dose distribution in depth. Such a homogeneous dose distribution over the target volume can be obtained by a superposition of many Bragg peaks with appropriate intensities and locations. These peaks will deliver a uniform dose in depth across the target volume, result in shaping the spread-out Bragg peak (SOBP) (ICRU, 2007). Clearly, the number of the appropriately weighted peaks changes by varying the extent of the target of interest. The methods used to produce SOBP can be categorized as: *a*) using materials in

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the beam path to shape the proton beam and to control the width and the energy of the beam (passive scattering), and *b*) using the magnetic beam scanning, in which magnetic fields sweep the proton beam over the desired area. Although the majority of proton therapy centers worldwide are based on utilizing scattered beams, the use of scanning beams are also of special interest in research facilities (Sawakuchi et al., 2010; Charlwood et al., 2016).

The distal margin of the SOBP is given by the distance between d_{20} and d_{80}, corresponding to the 20% and 80% of the maximum dose value. This quantity has been termed distal dose fall-off (DDF). However, the most clinically relevant parameters are the beam range and modulation width of the SOBP. The latter is defined as the distance in water between the distal and proximal 90% points of the maximum dose value (ICRU, 2007).

Experimentally, the mean projected range, denoted by R_0, can be measured either by employing a Faraday cup, or by using a dosimeter in the material under test (Gottschalk, 2012). In the first method, the range is considered as the depth at which half of the incident protons stop. Whilst, the range is considered as the depth in the target at the distal 80% (not exactly 80, but close enough) point of the Bragg peak, using a dosimeter. It is shown by $R_0 = d_{80}$.

Theoretically, the range of the protons can be calculated by obtaining the rate of energy loss per unit thickness of medium, *i.e.* the stopping power, and numerically integrating its inverse with respect to the energy. This quantity has been well described by Bethe and Bloch (Bethe, 1997; Bethe and Ashkin, 1953). However, the Bethe-Bloch equation can not be easily integrated analytically, and it is a time-consuming procedure. In evaluation of the Bragg curves through theoretical methods, which eventually results in range estimation, either Monte Carlo method or analytical approach can be employed. The latter uses a sufficiently accurate physical beam model which describes proton interactions by taking into account the physical processes occurring during the passage of protons through matter.

Arisen from undergoing a large number of collisions with atomic electrons, the incident protons lose their energy to the absorbing medium in small discrete amounts. This is a statistical process, and therefore the protons, even if their initial energy is exactly the same, will acquire an energy spread after passing through the medium. As a result, there would be a statistical fluctuation in the range of the incident particles, known as the range straggling (Bethe and Ashkin, 1953; Thomas, 1994). On the other hand, real proton beams have an initial energy distribution which depends on the characteristics of the accelerator, and on the beam shaping system. These uncertainties in proton radiotherapy can result in significantly compromised target coverage or normal-tissue sparing, which limit the full potential of the therapy. One of the most important problems in proton radiotherapy is therefore that how this straggling affects the range of protons in the target.

In this work, a highly-accurate analytical model has been used for dose evaluation in the target for proton beams of initial energy distribution. The resultant depth-dose curves are used for determining the theoretical range (d_{80}) in the medium, and for studying the effect of energy distribution on its value. The study is also extended to the case of stack of different materials, and surveying the accuracy of the range-energy approximation in presence of range straggling as well. The results obtained for different materials arrangements are compared with those of the Monte Carlo simulations with MCNPX code.

2 Materials and Methods

Range is defined as the depth at which half of the protons irradiated in the medium come to rest (Gottschalk, 2012). Rather than counting the number of protons, which can be accomplished employing a Faraday cup, the depth-dose distribution curves can also be used for measuring the range of the incident beam in the target. Although the shape of the Bragg curve is complex and depends on the energy spread and scattering properties of the delivery system, it is possible to be analytically described and hence the optimal weights for creating SOBP can be calculated. As a prominent example of the theories proposed, Bortfeld (Bortfeld, 1997) modeled the Bragg curve based on the power-law approximation which relates the initial energy E_0 to the range R_0 in the medium:

$$R_0 = \alpha E_0^p \tag{1}$$

where α and p are the parameters specified for each material. Owing to the statistical nature of the interaction of radiation with matter, actual ranges are distributed around a mean value which can be analytically calculated using Eq. (1). Considering this equation, the remaining energy, $E(z)$, at the arbitrary depth $z \leq R_0$ can be calculated using the following equation:

$$E(z) = \frac{1}{\alpha^{1/p}}(R_0 - z)^{1/p} \tag{2}$$

The Bortfeld theory represents the depth-dose employing the parabolic cylinder functions, incorporating both range straggling and initial energy distribution. This is a highly accurate model, verified by experiment, and it is applicable to the dose evaluation for normal incidence of a proton beam into a homogeneous target. The model has been extended to the oblique incidence of protons onto the target, and a uniform broad beam of an initial Gaussian angular distribution as well (Jette et al., 2010). Moreover, in recent works (Rasouli et al., 2015, 2016), general and mathematically easy to handle formulas have been presented to extend this model to the depth-dose evaluation in a desired compound or mixture composed of arbitrary number of constituent elements. In the next section, the range straggling for a number of materials has been studied. Moreover, we have used the Bortfeld model for dose evaluation in the target, for incident proton beams of clinical energy regime (3 to 300 MeV) with initial Gaussian energy distribution. In order to study the effect of energy straggling on the range, different widths of the distribution function have been considered.

Table 1: The values of m and κ for a number of arbitrary chosen materials. See Eq. (3).

Material	κ	m (MeV.cm$^{-\kappa}$)
Water	1.67	4.08 $\times 10^{-5}$
Bone	1.70	2.07 $\times 10^{-5}$
lexan	1.68	3.36 $\times 10^{-5}$
Lead	1.45	2.60 $\times 10^{-5}$

3 Results

3.1 Straggling in Range and Energy

Janni tables (Janni, 1982) report the range straggling for protons of energy between 1 keV to 10 GeV in elements with atomic number from 1 to 92, and in 63 compounds and mixtures. Figure 1 shows the range straggling, denoted by σ_R, for proton beams irradiated to four materials of interest in proton radiotherapy: water, bone, lexan, and lead. As figure shows, the value of σ_R depends on both the initial energy and the target material. The figure also justifies the reason that why water and lexan are employed in designing range modulator wheels.

By studying the values reported, it has been found that σ_R very closely obeys a power-law relation:

$$E_0 = m\sigma_R^{\kappa} \tag{3}$$

where m and κ are parameters which can be determined through a fit to the data reported in Janni (Janni, 1982) tables. Table 1 gives the values of these parameters calculated for the materials shown in Fig. 1 for protons with energies in the therapeutic range.

The straggling in the range observed is due to the statistical fluctuations in the amount of energy loss during the passage of protons into the target. For initial monoenergetic beam of 100 MeV, Fig. 2 shows the width of the energy distributions calculated in different depths of water using MCNPX 2.6 Monte Carlo simulations. As the figure shows, although the initial beam is monoenergetic, there is an energy straggling in depth, increases with increasing the penetration in the target.

3.2 Effect of energy distribution on the range

In addition to the range straggling occurs during the passage of protons through matter (Sec. 3.1), the real proton beams normally have an initial spectral energy distribution. This obviously imposes an extra variation on the penetration of protons into the target. In order to study the effect of this parameter on the shape of the Bragg curve, and consequently on the range, the proton beams with initial Gaussian energy distributions have been considered. Employing the model proposed by Bortfeld (Bortfeld, 1997), the depth-dose curves are evaluated for proton beams with different widths of energy distributions of 1 to 10 MeV, but with the same mean energy. Figure 3 shows some examples of these curves corresponding to the proton beams incident on water with initial mean energies of 50, 100, and 150 MeV.

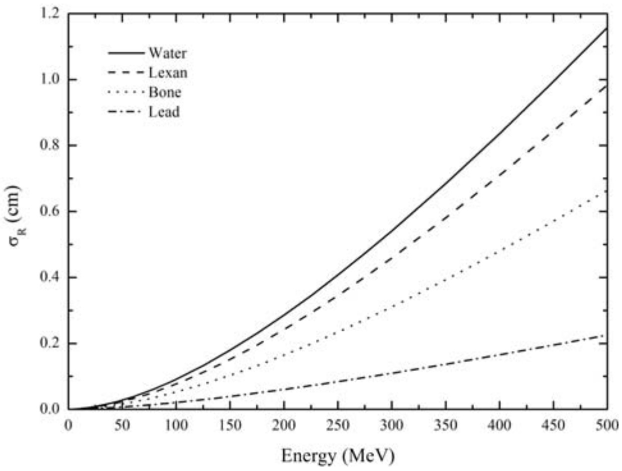

Figure 1: Range straggling in four materials of interest in proton radiotherapy. The data have been taken from Janni tables (Janni, 1982).

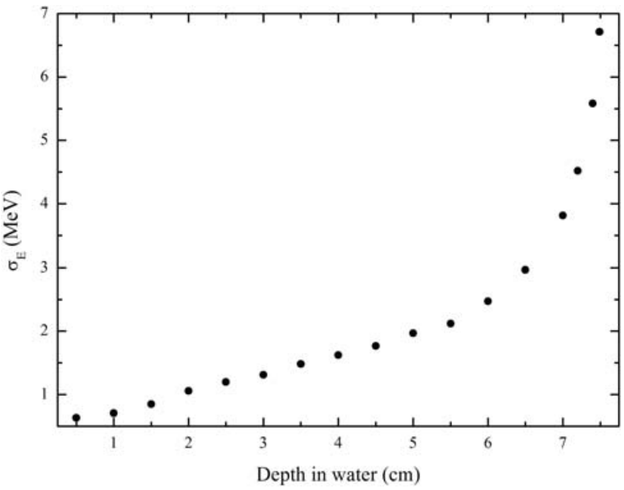

Figure 2: Width of the energy distributions calculated in different depths of water using MCNPX simulations for an initial monoenergetic proton beam of 100 MeV.

For these arbitrary chosen energies, the results show that the Bragg peaks' shifts due to enhancement of the width of energy distribution compared with the monoenergetic proton beam vary between about -0.25% to -25%. However, the most important result is that by increasing the energy spread of the proton beam, keeping the mean energy the same, the distal 80% of the Bragg peak, *i.e.* the range, remains unchanged. This is of high significance in solving the problem of practical importance in proton radiotherapy, *i.e.* range determination for transport of a proton beam through a stack of different materials, such as material arrangement in range modulator wheels, in nozzle design, and in different layers (structures) of the patient's body.

It has been considered a proton beam, either monoenergetic or with an initial energy distribution, entering an arrangement of materials designed to control and adjust the width and the energy of the beam to the values of interest, taking into account the tumor type, its shape, and its position in the patient's body. According to the data presented in Fig. 3, crossing the protons through the above system would increase the energy spread of the proton beam, but would not affect d_{80}. This section tests this assertion through estimating depth-dose distribution and calculating d_{80}. The penetration of protons in tissue then has been calculated using the repetitive use of Eq. (2). The results corresponding to examples of arrangement of materials have been reported in Table 2.

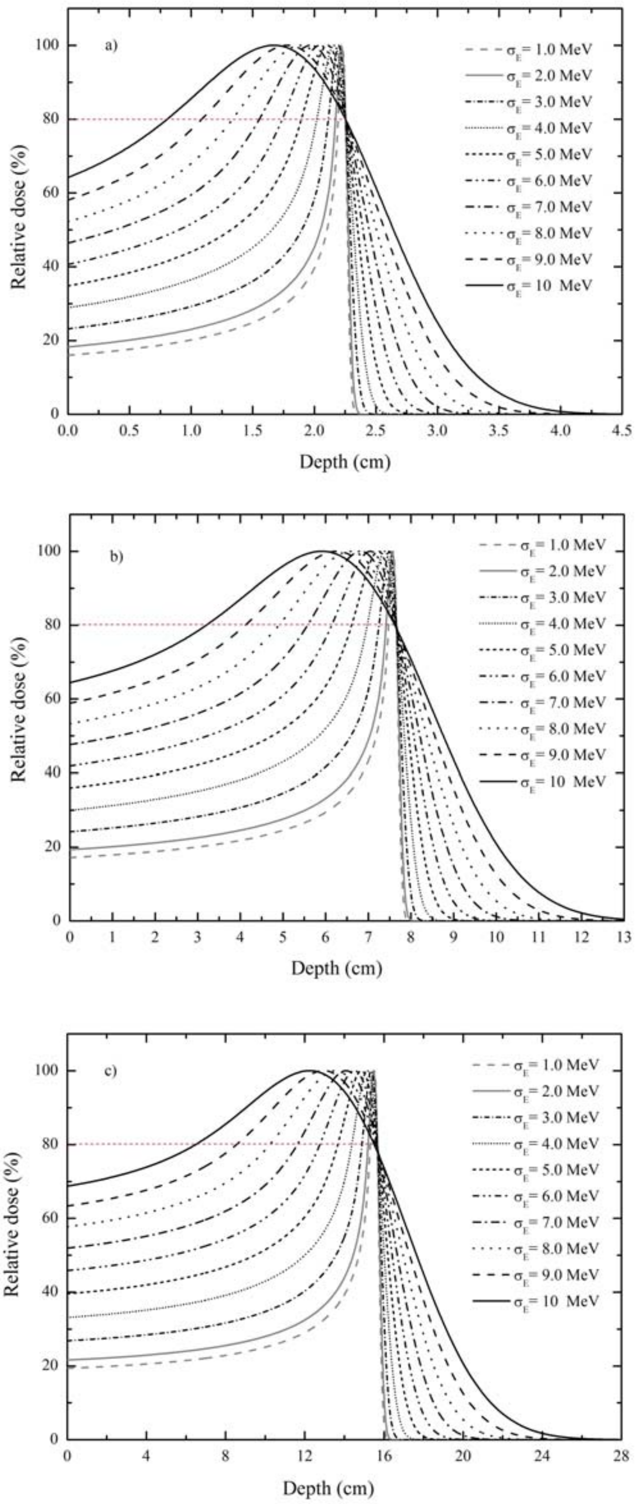

Figure 3: Bragg curves calculated using the Bortfeld model for proton beams with different widths of energy distribution for mean energies of a) 50 MeV, b) 100 MeV, c) 150 MeV, in water.

This table also gives the deviation from the values calculated using the Bragg curves evaluated by MCNPX from those of range-energy approximation.

4 Summary and Conclusions

The finite range and the Bragg peak of proton beam in the target have featured this particle as an appropriate candidate for radiation therapy. A full understanding of proton interactions with matter allows one to solve the two main physical problems that may arise in proton radiotherapy: designing beam lines, and predicting the dose distribution in the patient's body. The range of incident particles in the target is one of the main parameters that needs to be determined to handle the mentioned problems.

As Fig. 1 shows, even for a monoenergetic incident beam, all protons do not stop at the same depth. This spread in the stopping region, which increases in the case of initial energy spread of the incident beam, is called range straggling. Having performed a set of MCNPX simulations, it has been found that the more the protons penetrate in the target, the more the width of the energy distribution increases. However, the results corresponding to depth-dose curves predicted based on the Bortfeld model show that for different widths of beams with initial energy spectrum, keeping the mean energy the same, the distal 80% of the Bragg peak remains the same. This result is of great importance in one of the powerful approaches used for studying proton radiotherapy, *i.e.* analytical method.

Analytically, the mean proton range in the target can be calculated using the range-energy approximation (Eqs. (1) or (2)). However, the most important question was that if this relation can also be used for range determination in a stack of materials. Through analytical range determination for irradiated proton beams of different incident energies to stack of materials, including materials of practical importance in beam design and pre-clinical tests in proton radiotherapy, it was found that the mentioned formula gives acceptable results and is therefore still reliable. For the material arrangements reported in this work, the deviation from values for range calculated using Monte Carlo simulations from those of range-energy approximation is less than 1.7%. Analytically, accurate modeling of the corresponding depth-dose distribution in the presence of localized tissue inhomogeneities is presently an unsolved problem. There are a number of prominent papers which deal with transporting an arbitrary proton beam through a stack of different materials, and application of the Fermi-Eyges theory to transverse beam spreading in these media (Gottschalk, 2010; Goitein, 1978; Goitein and Sisterson, 1978; Jette et al., 1989; Jette and Walker, 1992). However, even if these studies can be useful for nozzle designs and beam shaping systems, owing to the fact that each slab must be homogeneous, can not of course be reliable for studying the beam transport and dose evaluation in real tissues which involve transverse heterogeneities. However, the presented study can be considered as a step toward solving this problem.

Table 2: Examples of values calculated for range using both MCNPX code and analytical formula in stacks of different materials and for different initial incident energies as well. The last column reports the deviation from the values calculated using the Bragg curves evaluated by MCNPX simulations from those of range-energy approximation (Eq. (2)).

No.	Material arrangement	E_0 (MeV)	MCNPX	Analytical	Deviation (%)
1	3 cm lexan + 2 cm water + 3 cm lexan + Pb	200	11.80	11.86	0.51
2	3 cm Pb + 2 cm lexan + 3 cm air + water	200	14.25	14.02	-1.61
3	3 cm Pb + 2 cm lexan + 3 cm air + 1 cm bone + water	200	13.55	13.32	-1.70
4	1 cm lexan + 1 cm Pb + 1 cm water + 1 cm air + 1 cm skin + 1 cm adipose + 2 cm bone + soft tissue	150	10.18	10.08	-0.98
5	6 cm water + 2 cm air + 1 cm skin + soft tissue	125	13.01	13.05	0.31
6	5 cm water + 2 cm air + 1 cm skin + 2 cm adipose + soft tissue	110	10.91	10.85	-0.55
7	3 cm water + 2 cm air + 1 cm water + 2 cm air + soft tissue	100	11.41	11.47	0.53
8	2 cm water + 1 cm lexan + 1 cm skin + soft tissue	80	4.86	4.88	0.41
9	1 cm bone + 1 cm adipose + 1 cm bone	60	2.21	2.22	0.45

References

Bethe, H. (1997). Theory of the passage of fast corpuscular rays through matter. In *Selected Works Of Hans A Bethe: (With Commentary)*, pages 77–154. World Scientific.

Bethe, H. A. and Ashkin, J. (1953). Passage of radiations through matter, Experimental Nuclear Physics E. Segre, 166–357.

Bortfeld, T. (1997). An analytical approximation of the bragg curve for therapeutic proton beams. *Medical Physics*, 24(12):2024–2033.

Charlwood, F. C., Aitkenhead, A. H., and Mackay, R. I. (2016). A monte carlo study on the collimation of pencil beam scanning proton therapy beams. *Medical Physics*, 43(3):1462–1472.

Goitein, M. (1978). A technique for calculating the influence of thin inhomogeneities on charged particle beams. *Medical Physics*, 5(4):258–264.

Goitein, M. and Sisterson, J. M. (1978). The influence of thick inhomogeneities on charged particle beams. *Radiation Research*, 74(2):217–230.

Gottschalk, B. (2010). On the scattering power of radiotherapy protons. *Medical Physics*, 37(1):352–367.

Gottschalk, B. (2012). Physics of proton interactions in matter, in: Proton Therapy Physics, Edited by Paganetti, H., CRC Press, Taylor Francis Group, BocaRaton.

ICRU (2007). Report No. 78, Prescribing, Recording, and Reporting Proton-Beam Therapy.

Janni, J. F. (1982). Energy loss, range, path length, time-of-flight, straggling, multiple scattering, and nuclear interaction probability: In two parts. Part 1. For 63 compounds Part 2. For elements 1≤ Z≤ 92. *Atomic Data and Nuclear Data Tables*, 27(4-5):341–529.

Jette, D., Lanzl, L. H., Pagnamenta, A., et al. (1989). Electron dose calculation using multiple-scattering theory: Thin planar inhomogeneities. *Medical Physics*, 16(5):712–725.

Jette, D. and Walker, S. (1992). Electron dose calculation using multiple-scattering theory: Evaluation of a new model for inhomogeneities. *Medical Physics*, 19(5):1241–1254.

Jette, D., Yuan, J., and Chen, W. (2010). Oblique incidence for broad monoenergetic proton beams. *Medical Physics*, 37(11):5683–5690.

Lu, H. and Flanz, J. (2012). Characteristics of clinical proton beams, in: Proton Therapy Physics, Edited by Paganetti, H., CRC Press, Taylor Francis Group, BocaRaton.

Rasouli, F. S., Masoudi, S. F., and Jette, D. (2015). On the proton range and nuclear interactions in compounds and mixtures. *Medical Physics*, 42(5):2364–2367.

Rasouli, F. S., Masoudi, S. F., and Jette, D. (2016). On the analytical proton dose evaluation in compounds and mixtures. *Medical Physics*, 43(1):303–307.

Sawakuchi, G. O., Mirkovic, D., Perles, L. A., et al. (2010). An mcnpx monte carlo model of a discrete spot scanning proton beam therapy nozzle. *Medical Physics*, 37(9):4960–4970.

Thomas, E. (1994). Stopping powers and ranges for protons and alpha particles. *HEALTH PHYSICS*, 66:352–352.